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Freeze-out of Weak Interactions:

As we saw, weak interactions cannot keep out with the expansion of the universe and drop out of equilibrium at temperatures $T < O(\text{MeV})$:

$$\Gamma_{weak} \sim G_F^2 T^5 , \quad H \sim \frac{T^2}{M_P}$$

$$T < O(\text{MeV}) \Rightarrow \Gamma_{weak} < H$$

This has important consequences. Here we discuss two of them in more detail.

Neutrino Decoupling:

Neutrinos have only weak interactions. They can stay in thermal equilibrium with other elementary particles (photons, electrons/positrons) through weak interaction. However, this can only happen down to temperatures $T \sim O(\text{MeV})$. At $T < 1$ MeV, the primordial plasma consists of two components: (1) Those with electromagnetic interactions (photons, electrons/positrons), (2) Those with weak interactions (neutrinos and antineutrinos). These two components do not communicate with each other at $T < 1$ MeV as neutrinos decouple due to inefficiency of weak interactions.

Soon after decoupling of the neutrinos, electrons/positrons become non-relativistic. This occurs when $T < m_e \simeq 0.5$ MeV. Above this temperature pair creation $\gamma\gamma \to e^- e^+$ and pair annihilation $e^- e^+ \to \gamma\gamma$ happen efficiently. At $T < m_e$ pair creation becomes kinematically suppressed. Pair annihilation, however, proceeds efficiently due to its electromagnetic nature. As a result, essentially of the electrons/positrons disappear. The $e^- e^+ \to \gamma\gamma$ is an adiabatic process because of its efficiency (i.e., $\Gamma_{ann} \gg H$). The total entropy of the universe must therefore be conserved during this process. More precisely:

$S_{after} = S_{before}$ $\qquad S_{after} = s_{after}\, a^3_{after}$, $S_{before} = s_{before}\, a^3_{before}$

Here "after" and "before" refer to two moments of time sufficiently after pair annihilation completion and before its start respectively:

before: $T > m_e$ $\qquad$ after: $T \ll m_e$

We note that:

$s_{before} = \frac{2\pi^2}{45} g_{*s,before} T^3_{before}$ $\qquad g_{*s,before} = 2 + \frac{7}{8} \times 4 = \frac{11}{2}$

$s_{after} = \frac{2\pi^2}{45} g_{*s,after} T^3_{after}$ $\qquad g_{*s,after} = 2$

Here g_* counts for the number of relativistic degrees of freedom with electromagnetic interactions. Since neutrinos have already decoupled from photons and electrons/positrons, they will not learn about the pair annihilation.

Then $S_{after} = S_{before}$ results in:

$$g_{*,before}\, T^3_{before}\, a^3_{before} = g_{*,after}\, T^3_{after}\, a^3_{after} \Rightarrow$$

$$\frac{T_{after}}{T_{before}} = \left(\frac{11}{4}\right)^{1/3} \frac{a_{before}}{a_{after}}$$

If the universe was just expanding, then $\frac{T_{after}}{T_{before}} = \frac{a_{before}}{a_{after}}$.

However, pair annihilation transfers the energy in e^- and e^+ to photons in adiabatic fashion. This results in heating up of the photons and yields the factor $\left(\frac{11}{4}\right)^{1/3}$. The comoving temperature of photons increases by this factor after the completion of pair annihilation. However, the comoving temperature of neutrinos remains constant because of their decoupling from the plasma. As a result, at $T \ll m_e$, we have:

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \qquad (I)$$

This implies that at $T \ll m_e$ the primordial plasma consists of two components (photos and neutrinos/antineutrinos) that have different temperatures. The energy density in radiation at $T \ll 1$ MeV ($t \gg 1$ sec) thus follows;

$$\rho_r = \frac{\pi^2}{30} \times 2 \times T_\gamma^4 + \frac{\pi^2}{30} \times \left(\frac{7}{8} \times 6\right) \times T_\nu^4 = \frac{\pi^2}{30}\left[2 + \frac{7}{8} \times 2 \times 3 \times \left(\frac{4}{11}\right)^{\frac{4}{3}}\right] T_\gamma^4$$

In general, additional relativistic particles beyond the standard model can also contribute to ρ_r. A convenient parameterization that takes this contribution into account is as follows;

$$\rho_r = \frac{\pi^2}{30}\left[2 + \frac{7}{8} \times 2 \times N_\nu^{eff} \times \left(\frac{4}{11}\right)^{\frac{4}{3}}\right] T^4 \qquad (\mathrm{II})$$

Here T refers to the temperature of photons, and N_ν^{eff} includes the contribution from all weakly (and superweakly) interacting light particles. In the case of the standard model, the only such particles are the three neutrino flavors. Actually, $N_\nu^{eff} = 3.04$, where the slight deviation from 3 has to do with the fact that neutrinos can still interact with electrons/positrons, although inefficiently, at $T < 1$ MeV. This residual interactions increases T_ν after the completion of pair annihilation slightly thus resulting in $N_\nu^{eff} = 3.04$.

The epoch of matter-radiation equality is found by equating the expression for ρ_r in Eq.(II) and the energy density in matter ρ_m. As we will see later, when the equality happens t_{eq} affects the power spectrum of the CMB. One can therefore constrain t_{eq} from CMB data, which leads to a constraint on N_ν^{eff}. We will discuss this in more detail later on.

Neutron to Proton Ratios

Another important consequence of the freeze-out of weak interactions is on the ratio of the number of neutrons to protons. This is a very important parameter in determining the relative abundance of primordial ^4He and H, which can be inferred observationally.

For a closer look, let us consider various weak interactions between neutrons and protons:

$n \rightleftarrows p + e^- + \bar{\nu}_e$ $\quad m_n \simeq 939.57$ MeV , $m_p \simeq 938.27$ MeV

$p + e^- \rightleftarrows n + \nu_e$ $\quad \Delta E \simeq 0.8$ MeV $\quad$ (III)

$p + \bar{\nu}_e \rightleftarrows n + e^+$ $\quad \Delta E \simeq 1.8$ MeV

At $T \gtrsim 1$ MeV, all these 6 reactions occur efficiently (i.e., at rates faster than the expansion rate of the universe). As a result, the neutron to proton ratio $\frac{n}{p}$ is given by its equilibrium value:

$$\frac{n}{p} = \exp\left(-\frac{m_n - m_p}{T}\right)$$

Here we have used the fact that $E_n - E_p = m_n - m_p$ at $T \ll 1$ GeV. We have also assumed zero chemical potential for neutrinos ($\mu_{\nu_e} = 0$). One can show that the chemical potential of electrons μ_e is negligible anyways.

The situation changes once $T < 1$ MeV. At such low temperatures weak interactions between two particles becomes inefficient. Moreover, thermal energy is not large enough to make some reactions (like $p + e^- + \bar{\nu}_e \to n$) kinematically possible. Out of the 6 reactions in Eq. (III), only the neutron decay can proceed uninhibited:

$$n \to p + e^- + \bar{\nu}_e \qquad T < 1\,\text{MeV}$$

At the time weak interactions freeze, we have:

$$\left(\frac{n}{p}\right)_0 \simeq \exp\left(-\frac{m_n - m_p}{1\,\text{MeV}}\right) \sim \frac{1}{6} \qquad (\text{IV})$$

At later times, $t > 1$ sec, this number further decreases because of the neutron decay:

$$\left(\frac{n}{p}\right)_{(t)} = \left(\frac{n}{p}\right)_0 \exp\left(-\frac{t - 1\,\text{sec}}{\tau_n}\right) \qquad (\text{V})$$

Here $\tau_n \simeq 881$ sec is the neutron lifetime.

Had the neutrons been free, they would have completely decayed according to Eq. (V) thus leading to a universe with protons and electrons (by the charge neutrality argument) only. This is certainly very different from the universe we observe. In fact, neutrons and protons bind to form light nuclei, at which point neutron decay according to Eq. (V) ceases. This process is called Big-Bang Nucleosynthesis (BBN), which is the topic of next few lectures.